

FINAL EXAMINATION - DAWSON COLLEGE
Applied Mathematics (LABORATORY TECHNOLOGY - ANALYTICAL CHEMISTRY)

201-923-DW
Fall 2019
Final Examination
December 17th, 2019
Time Limit: 3 hours

Name: _____

ID#: _____

Instructor: **Emilie Richer**

- This exam contains 15 pages (including this cover page) and 17 problems. Check to see if any pages are missing.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page, and please indicate that you have done so.
- Give the work in full; – unless otherwise stated, reduce each answer to its simplest, exact form; – and write and arrange your exercise in a legible and orderly manner.
- You are only permitted to use the **Sharp EL-531XG** or **Sharp EL-531X** calculator.
- This examination booklet must be returned intact.
- Good luck!

| Question | Points | Score |
|----------|--------|-------|
| 1 | 5 | |
| 2 | 8 | |
| 3 | 5 | |
| 4 | 6 | |
| 5 | 6 | |
| 6 | 12 | |
| 7 | 4 | |
| 8 | 4 | |
| 9 | 6 | |
| 10 | 6 | |
| 11 | 8 | |
| 12 | 7 | |
| 13 | 6 | |
| 14 | 4 | |
| 15 | 4 | |
| 16 | 3 | |
| 17 | 6 | |
| Total: | 100 | |

Question 1. (5 marks) Given the matrices:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 0 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 2 \\ 2 & -1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 & 2 \\ 0 & -1 & 0 \\ 3 & 1 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 4 & -7 \\ 3 & 1 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 3 & 2 \end{bmatrix}$$

Find the following (when possible):

(a) $A + B$

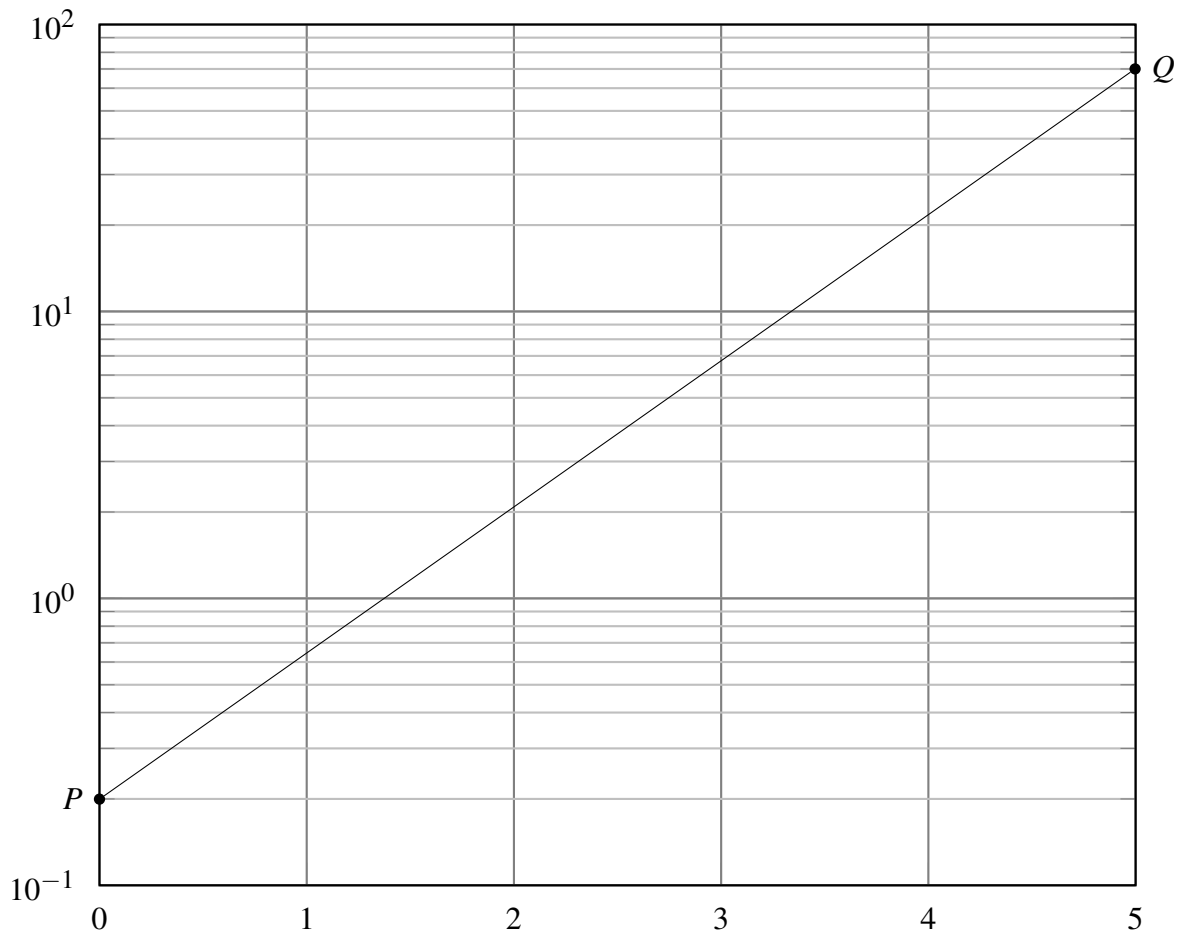
(b) B^{-1}

(c) E^t

(d) $-D - 2B$

(e) CA

Question 2. (8 marks) Consider the function $y = f(x)$ whose graph, containing the points P and Q , is illustrated below:



- Give the coordinates of points P and Q
- Use the points P and Q to find a formula for the function $y = f(x)$.
- Graph the function $y = g(x) = 2^{x-2}$ on the same graph.

Question 3. (5 marks) For each of the following matrices state whether they are in row-echelon form **REF**, reduced row-echelon form (**RREF**) or neither (**N**). If you answer neither, give an explanation.

(a)
$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

(c)
$$\begin{bmatrix} 1 & 2 & -9 & 7 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

(d)
$$\begin{bmatrix} 1 & 0 & -9 & 7 & 3 \\ 0 & 1 & 1 & 2 & 1 \end{bmatrix}$$

(e)
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Question 4. (6 marks) The following three augmented matrices are in row echelon form. For each matrix, determine the solution(s) of the corresponding system of linear equations or state that there is no solution.

a.
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

b.
$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 2 & 1 \\ 0 & 0 & 1 & -1 & 5 \end{array} \right]$$

c.
$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -3 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

Question 5. (6 marks) Consider the following system of equations:

$$\begin{aligned}x + z &= 1 \\x - y + z &= 3 \\x + y - z &= 2\end{aligned}$$

- (a) Express the system in the form of a matrix equation $Ax = b$.
- (b) **Use the inverse** of a suitable matrix to give the solutions to this system.

Question 6. (12 marks) Solve for θ in the following equation, giving all solutions in the range $0^\circ \leq \theta < 360^\circ$ and giving exact solutions where possible. Give your solutions in **both degree and radian measurement**.

(a) $3 \cos \theta - 4 \cos^2 \theta = 0$

(b) $\sqrt{3} \sec \theta + 2 = 0$

(c) $1 - 9 \tan \theta = 0$

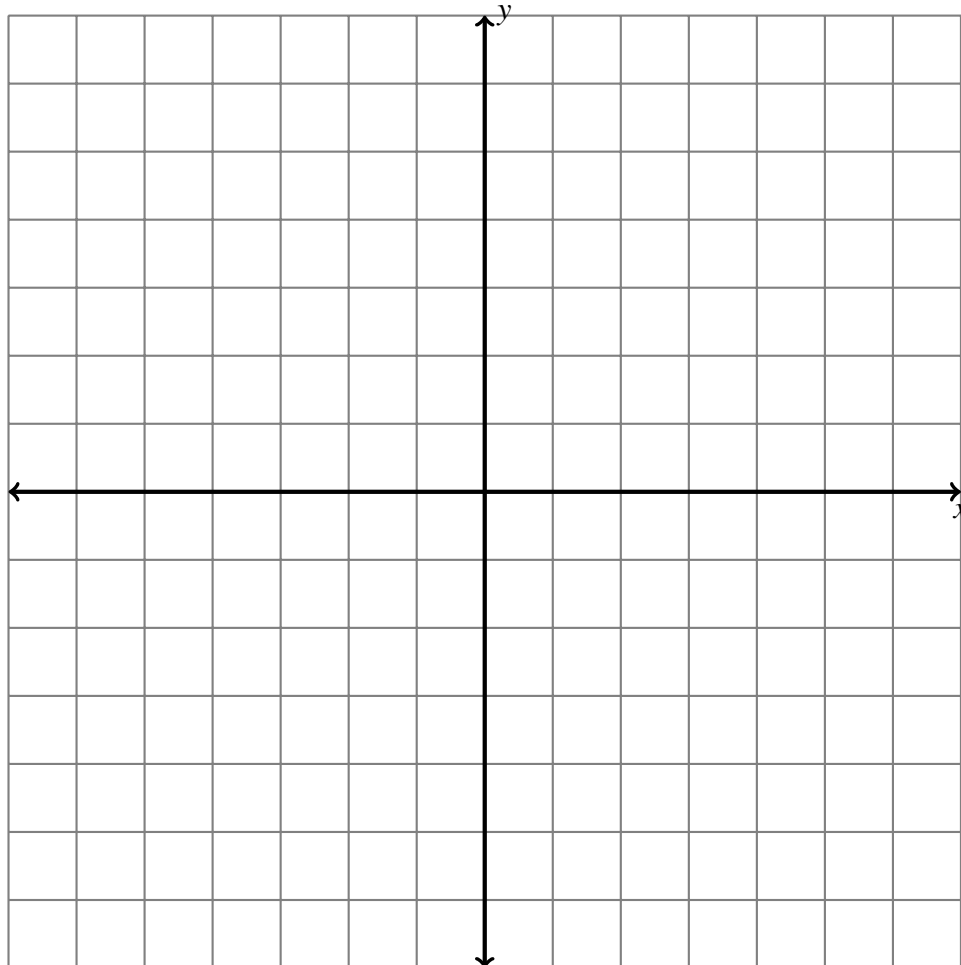
Question 7. (4 marks) Find the determinant of the following matrix: $\begin{bmatrix} -2 & 0 & 1 \\ 2 & -1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$

Question 8. (4 marks) Give the x and y components of the vector \vec{v} that has magnitude 1205 and direction 212° .

Question 9. (6 marks) (a) Graph the function $f(x) = 3^{(x+3)} - 4$, making sure that

- the general shape of the function is clearly illustrated
- all asymptotes are indicated
- at least three distinct points are labelled.

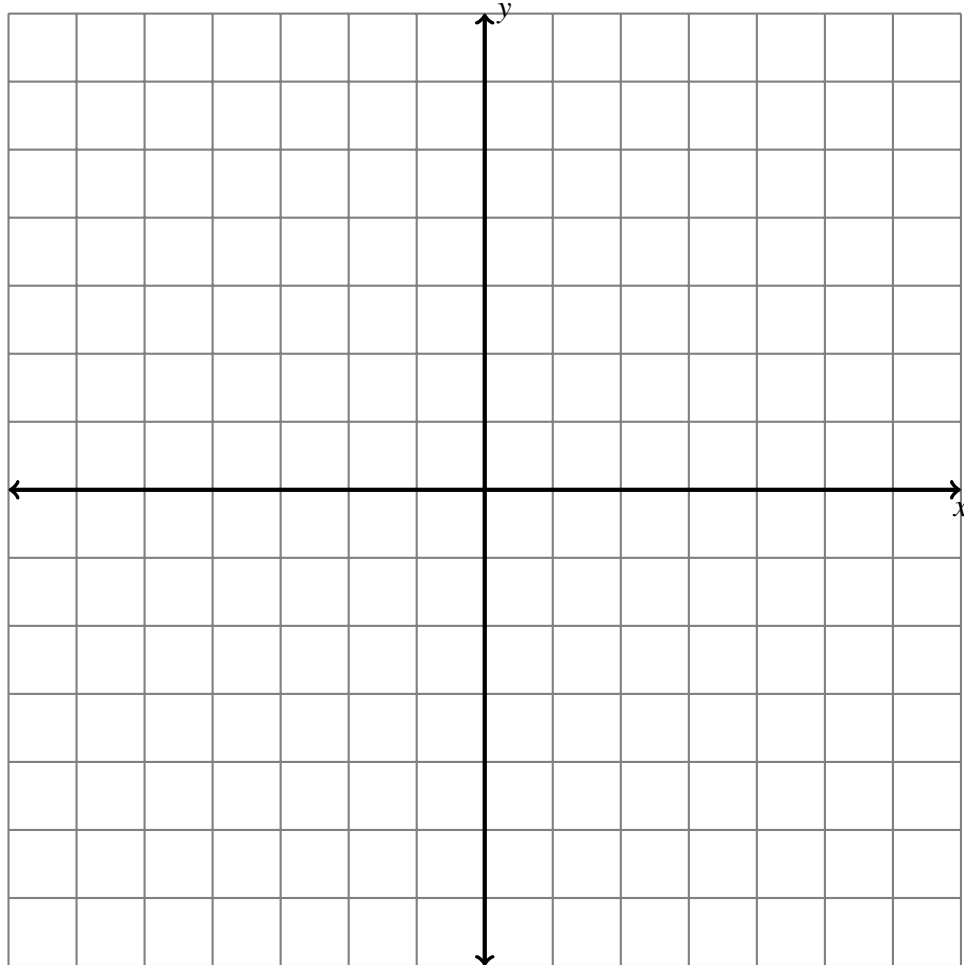
(b) State the domain and range of $f(x)$



Question 10. (6 marks) (a) Graph the function $g(x) = \log_2(4 - x) + 2$, making sure that

- the general shape of the function is clearly illustrated
- all asymptotes are indicated
- at least three distinct points are labelled.

(b) State the domain and range of $g(x)$



Question 11. (8 marks) Solve for x in each of the following equations:

(a) $2^{(x+3)} = 128$

(b) $4 = 3(2^{0.1x})$

(c) $\log_x\left(\frac{1}{36}\right) = 2$

(d) $\ln(1-x) = 2$

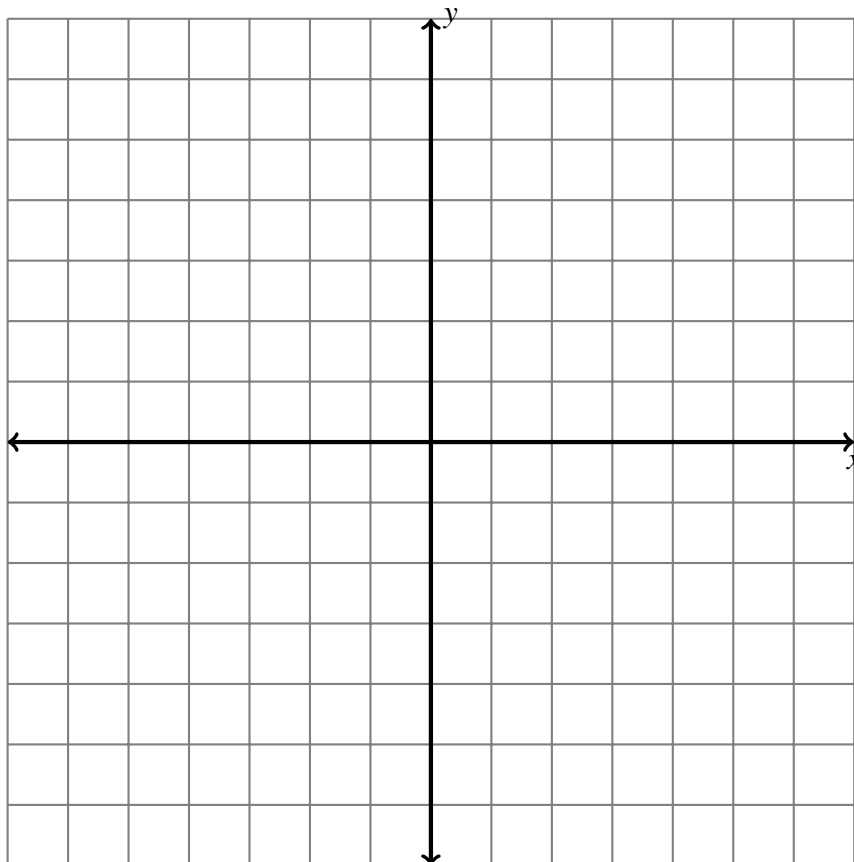
Question 12. (7 marks) Consider the following points $P(1, -1)$, $Q(2, 4)$ and $R(-3, -2)$. Find the following vectors, then draw them in **standard position** on the same grid below:

i. $\vec{a} = \vec{PQ}$

ii. $\vec{b} = \vec{RQ}$

iii. $\vec{c} = -2\vec{a} + \vec{b}$

iv. Give the direction and magnitude of \vec{c}



Question 13. (6 marks) Solve for x in each of the following equations:

(a) $4^{(2x-1)} = 7^{(x+2)}$

(b) $2\log(x-3) - \log(x-1) = 0$

Question 14. (4 marks) Solve the right angle triangle ABC (where $C = 90^\circ$) given: $B = 42^\circ$ and $a = 2$.

Question 15. (4 marks) Show how you would evaluate the following **without** a calculator.

(a) $\frac{\log_8 x}{\log_2 x}$

(b) $4\ln(\sqrt{x}) - 2\ln x$

Question 16. (3 marks) Evaluate the following **with** a calculator (show your steps in order to get part marks).

(a) $2\log_2(7)$

(b) $e^{(-\log_3 4)}$

Question 17. (6 marks) Graph the following function:

$$f(x) = \begin{cases} x+2 & x \geq -1 \\ -2x^2+4 & x < -1 \end{cases}$$

Make sure that the following are clearly illustrated:

- the general shape of the function
- the behaviour of the function at $x = -1$
- four different points, clearly labelled

