<u>201-NYA-05-04</u>
Calculus for Lab Tech

FINAL EXAM
WINTER 2021

1. **[12 Pts]** Evaluate the following limits using the appropriate technique or theorem. Show all your work.

   a)
$$\lim_{x\to 1}\frac{\frac{x}{4}-\frac{1}{x+3}}{x-1}$$

   b)
$$\lim_{x\to\infty}\frac{x^3-3x^2+15}{17+4x-8x^3}$$

   c)
$$\lim_{x\to 0}\frac{-2\sin 3x}{x}$$

   d)
$$\lim_{x\to\infty}\left((x^2+x)\cos\left(\frac{1}{x}+2\right)\right)$$

2. Consider the piece-wise function defined below.

$$f(x)=\begin{cases}\dfrac{x^2-1}{x+1} & x<-1\\ k & x=-1\\ 2x^3 & x>-1\end{cases}$$

   a) **[4 Pts]** Find the limit, if it exists:
$$\lim_{x\to -1} f(x)$$

   b) **[2 Pts]** Find the value of k that makes the function $f(x)$ continuous at $x=-1$. Justify your answer.

3. **[6 Pts]** Use <u>**only**</u> the limit definition (the four-step process) to find the derivative of:

$$f(x) = x^2 - 3x + 18$$

4. **[9 Pts]** Use the appropriate rules to find the derivative of the following functions. Do NOT simplify your answer.

   a)
   $$y = (\tan 5x - \csc 3x)^8$$

   b)
   $$y = \frac{\cos(x^7 - 7)}{7^x + 17}$$

   c)
   $$y = (1 + \cot x)^{2+x^2}$$

5. **[4 Pts]** Find the equation of the normal line to the function:

$$y = \sin\left(\sqrt{x} + \frac{\pi}{2}\right) \; at\ the\ point \; \left(\frac{\pi^2}{4}, 0\right)$$

6. **[5 Pts]** Find the derivative y' for the implicitly defined function:

$$x^2 y - \sqrt{\pi} = e^y$$

7. **[5 Pts]** A police officer travelling north at 40 km/hr sees a car speed through an intersection, driving east. Using his radar gun, he measures a reading of 30 km/hr. By using landmarks, he judges himself to be 0.3 km from the intersection of the two roads, and the car to be 0.4 km to the right of that same intersection. If the other road has a speed limit of 50 km/hr, is the other driver speeding? Justify your answer mathematically.

8. **[3 Pts]** A toy manufacturer makes cubic building blocks for babies. The edge of a block is supposed to be 5 inches long, however the measurements can be off by $\pm$ 0.2 inches due to manufacturing. Use differentials to find the *propagated error* in the volume of the building block.

9. **[6 Pts]** You are staying at a resort and want to visit a secluded island off the coast. The diagram below illustrates all the distances. Your transportation options are a land taxi at 1$/km, and a water taxi at 5$/km. What distance should you cover with the water taxi in order to **minimize** your total transportation cost?

10. Consider the function:

$$f(x) = x^2 - \frac{2}{x}$$

a) **[1 Pt]** Find the horizontal and vertical asymptotes, if any.

b) **[5 Pts]** Determine the intervals where $f(x)$ is increasing, where it is decreasing, and identify any relative extrema.

c) **[5 Pts]** Determine the intervals where $f(x)$ is concave up, where it is concave down, and identify any inflection points.

d) **[2 Pts]** Sketch the function using the above information and extra points as needed.

11. **[6 Pts]** Consider the function

$$f(x) = x^2 + 1$$

a) Find a value $c \in [0,3]$ that satisfies the Mean Value Theorem for <u>differentiation</u>.
b) Find a value $c \in [0,3]$ that satisfies the Mean Value Theorem for <u>integration</u>.

12. **[3 Pts]** Given $\int_1^2 f(x)dx = 6$, $\int_1^5 f(x)dx = -4$ and $\int_1^5 g(x)dx = 3$, calculate:

$$\int_2^5 f(x)dx + \int_1^5 \left(\frac{1}{2}f(x) - 3g(x)\right)dx$$

13. **[4 Pts]** Integrate:

a)

$$\int \left(13\sec x \tan x - \frac{2}{\sqrt{3}}\csc^2 x\right) dx$$

b)

$$\int (-8\sec^2 x + 5\csc x \cot x)\, dx$$

14. **[5 Pts]** Use **<u>only</u>** the limit of Riemann Sums, and the right end-point, to evaluate:

$$\int_0^2 (x^3 - 1)dx$$

Reminder:

$$\sum_{k=1}^{n} c = c \cdot n \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

15. **[4 Pts]** The concentration of a solution t seconds after the addition of a solute is given by:

$$C(t) = \frac{3}{t} + \frac{2}{\sqrt{t}} + 1 \quad moles/L$$

What is the average concentration of the solution from $t = 1$ to $t = 4$ seconds?

16. **[3 Pts]** The velocity of a certain particle is given by:

$$v(t) = 7\sin t + 5\cos t \quad ft/sec$$

Find the position function $s(t)$ if $s(0) = 0$.

17. **[2 Pts]** Use Part 1 of the Fundamental Theorem of Calculus to find $F'(x)$ is:

$$F(x) = \int_{\sec x}^{e} \sqrt{t^3 + 5}\ dt$$

18. **[3 Pts]** Rounding off your answer to 3 decimal places, calculate the area of the region bounded above by

$$f(x) = 1 + e^x$$

bounded below by

$$g(x) = -\frac{2}{x^2}$$

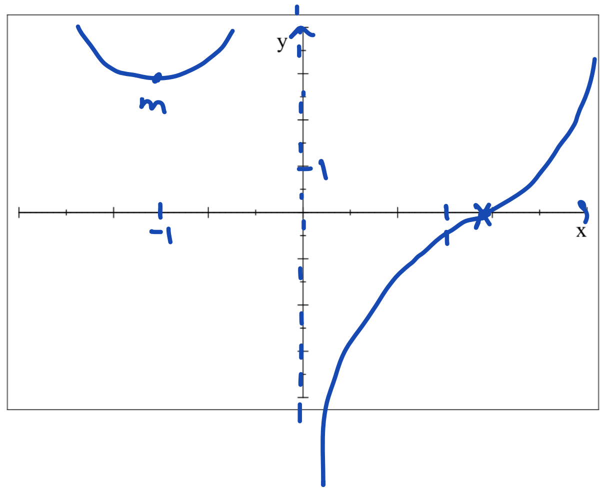
and between the lines $x = -1$ and $x = 1$.

Answers:

1.
    a. $\frac{5}{16}$
    b. $\frac{-1}{8}$
    c. -6
    d. 0
2.
    a. -2
    b. -2
    c. $k = -2$
3. $f'(x) = 2x - 3$
4.
    a. $y' = 8(\tan 5x - \csc 3x)^7(5\sec^2 5x + 3\csc 3x \cot 3x)$
    b. $y' = \frac{-7x^6 \sin(x^7-7)(7^x+17)-(\ln 7)7^x \cos(x^7-7)}{(7^x+1\ \ )^2}$
    c. $y' = \left(2x\ln(1+\cot x) - \frac{(\csc^2 x)(2+x^2)}{1+\cot}\right)(1+\cot x)^{2+x^2}$
5. $y = \pi x - \frac{\pi^3}{4}$
6. $y' = \frac{-2xy}{x^2-e^y}$
7. Yes, because $x' = 67.5\ km/h > 50\ km/h$
8. $dV = \pm 15\ in^3$
9. $z = 6.12\ km$
10.
    a. No horizontal asymptote, and one vertical asymptote $x = 0$.
    b. Increasing on $(-1,0) \cup (0,\infty)$, decreasing on $(-\infty,-1)$, and one relative minimum at $(-1,3)$.

c. Concave up on $(-\infty, 0) \cup (\sqrt[3]{2}, \infty)$, concave down on $(0, \sqrt[3]{2})$, and one inflection point at $(\sqrt[3]{2},0)$.

d.



11.

a. $c = \frac{3}{2}$

b. $c = \sqrt{3}$

12. -21

13.

a. $13\sec x + \frac{2}{\sqrt{3}}\cot x + C$

b. $-8\tan x - 5\csc x + C$

14. 2

15. $\ln 4 + \frac{7}{3} \sim 3.72\, moles/L$

16. $s(t) = -7\cos t + 5\sin t + 7$

17. $F'(x) = -\sqrt{\sec^3 x + 5}(\sec x \tan x)$

18. $A \sim 0.35\, units^2$