DAWSON COLLEGE
MATHEMATICS DEPARTMENT

Final Examination
Winter 2022

Calculus for Lab Tech
201-NYA-05 section 04

Date: Wednesday, May 25th, 2022 at 2pm
Instructor: C. Farnesi

1. **[12 Pts]** Evaluate the following limits using the appropriate technique or theorem. Show all your work.

a)
$$\lim_{x\to 2}\frac{\frac{x}{3}-\frac{2}{x+1}}{x-2}$$

b)
$$\lim_{x\to -\infty}\frac{2x^4-x^3+15x}{17+4x^2-9x^4}$$

c)
$$\lim_{x\to 0}\frac{5\sin 7x}{x}$$

d)
$$\lim_{x\to -1}\left((x^2-1)\cdot\cos\left(\frac{1}{x+1}\right)\right)$$

2. **[6 Pts]** Consider the piece-wise function defined below.

$$f(x)=\begin{cases}\dfrac{x^2-8}{2} & x<3\\ k & x=3\\ \dfrac{3+\sqrt{x-3}}{x+3} & x>3\end{cases}$$

a) Find the limit, if it exists:

$$\lim_{x \to 3^+} f(x)$$

b) Find the limit, if it exists:

$$\lim_{x \to 3} f(x)$$

c) Use the conditions of continuity to find the value of k that makes the function $f(x)$ continuous at $x = 3$.

3. **[6 Pts]** Use **only** the limit definition (the four-step process) to find the derivative of:

$$f(x) = 2x^2 + 3x - 45$$

4. **[9 Pts]** Use the appropriate rules to find the derivative of the following functions. Do NOT simplify your answer.

a)

$$y = 7^x\left(e^{7\sin x}\right)$$

b)

$$y = \frac{\arctan 3x}{\tan 3x}$$

c)

$$y = (x^2 + 1)^{\csc 4x}$$

5. **[4 Pts]** Find the equation of the normal line to the function:

$$y = \cos\left(\frac{1}{x}\right) - \frac{8}{\pi^3}$$

at the point $\left(\frac{2}{\pi}, -\frac{8}{\pi^3}\right)$.

6. **[4 Pts]** Find the derivative y' for the implicitly defined function:

$$x^4 y = 16x + y^2$$

7. **[4 Pts]** Two boats leave port at the same time. One boat goes north at 52 miles/hr, and the second goes east at 44 miles/hr. How fast is the distance between the two boats changing when the first boat is 40 miles north from port, and the second is 30 miles east from port?

8. **[6 Pts]** You are given a 10" x 10" piece of cardboard. You are then told to cut identical squares in each corner and fold the resulting flaps to form an open box. Find the dimensions of the box that gives a maximum volume.

9. Consider the function:

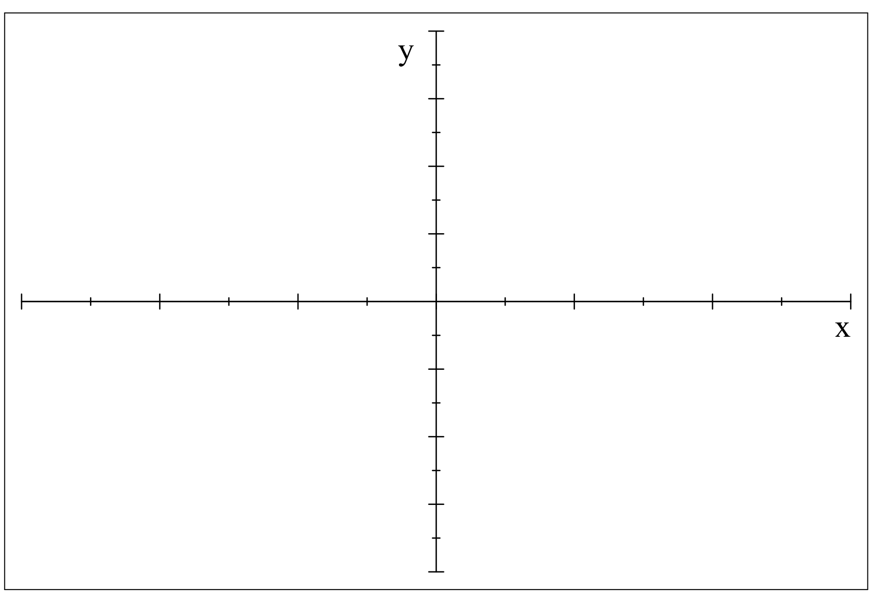
$$f(x) = x^2 + \frac{2}{x}$$

   a) **[2 Pt]** Find the horizontal and vertical asymptotes, if any.

   b) **[5 Pts]** Determine the intervals where $f(x)$ is increasing, where it is decreasing, and identify any relative extrema.

   c) **[5 Pts]** Determine the intervals where $f(x)$ is concave up, where it is concave down, and identify any inflection points.

   d) **[2 Pts]** Sketch the function using the above information and extra points as needed. Scale each axis accordingly so that you use up as much space as possible.



10. **[6 Pts]** Consider the function

$$f(x) = \frac{4}{\sqrt{x}}$$

a) Find a value $c \in [1,4]$ that satisfies the Mean Value Theorem for <u>differentiation</u>.

b) Find a value $c \in [1,4]$ that satisfies the Mean Value Theorem for <u>integration</u>.

11. **[3 Pts]** A company makes chemistry lab equipment, including round-bottom flasks. The sphere part of these round-bottom flasks is supposed to have a radius of 2 inches, but the measurements can be off by $\pm$ 0.05 inches due to manufacturing. Use differentials to find the *propagated error* in the volume of the sphere of these flasks. <u>Reminder</u>: $V = \frac{4}{3}\pi r^3$.

12. **[3 Pts]** Given $\int_1^3 f(x)dx = 5$, $\int_1^8 f(x)dx = -13$ and $\int_1^8 g(x)dx = 21$, calculate:

$$\int_3^8 f(x)dx + \int_8^1 g(x)dx + \int_1^8 \left(2f(x) - \frac{1}{7}g(x)\right)dx$$

13. **[2 Pts]** Integrate:

$$\int \left(23\csc^2 x - \frac{3}{\sqrt{2}}\sec x \tan x\right)dx$$

14. **[7 Pts]** Use **<u>only</u>** the limit definition of the Definite Integral, and the right end-point, to evaluate:

$$\int_0^3 (x^3 + 1)dx$$

Reminder:

$$\sum_{k=1}^{n} c = c \cdot n \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

15. **[4 Pts]** The concentration of a solution t seconds after the addition of a solute is given by:

$$C(t) = \frac{t^2}{8} + \frac{2}{\sqrt[3]{t}} \quad moles/L$$

What is the average concentration of the solution from $t = 1$ to $t = 8$ seconds? Round your answer to 3 decimal places.

16. **[4 Pts]** The velocity of a certain particle is given by:

$$v(t) = 3e^t - 2\sin t \quad ft/sec$$

Find the position function $s(t)$ if $s(0) = 0$.

17. **[2 Pts]** Use Part 1 of the Fundamental Theorem of Calculus to find $F'(x)$ if:

$$F(x) = \int_2^{(3x+5)^2} \arcsin t \, dt$$

18. **[4 Pts]** Rounding off your answer to 3 decimal places, calculate the area of the region bounded above by

$$f(x) = x^2 + 2$$

bounded below by

$$g(x) = \frac{3}{x}$$

and between the lines $x = 1$ and $x = e$.

Answers:

1.
   a. 5
   b. $\frac{-2}{9}$
   c. 35
   d. 0
2.
   a. 0.5

b. 0.5

c. $k = 0.5$

3. $f'(x) = 4x + 3$

4.

a. $y' = (7^x \ln 7)e^{7\sin x} + \left(-7\cos x\, e^{7\sin x}\right)(7^x)$

b. $y' = \dfrac{\frac{3}{1+(3x)^2}\tan 3x - (3\sec^2 3x)\arctan 3}{(\arctan 3x)^2}$

c. $y' = \left(-4\csc 4x \cot 4x \ln(x^2+1) + \frac{2x}{x^2+1}\csc 4x\right)(x^2+1)^{\csc 4x}$

5. $y = -\frac{4}{\pi^2}x$

6. $y' = \frac{16-4x^3y}{x^4-2y}$

7. $z' = 68\ miles/hr$

8. The dimensions are $\frac{20}{3}\ in$ by $\frac{20}{3}\ in$ by $\frac{5}{3}\ in$

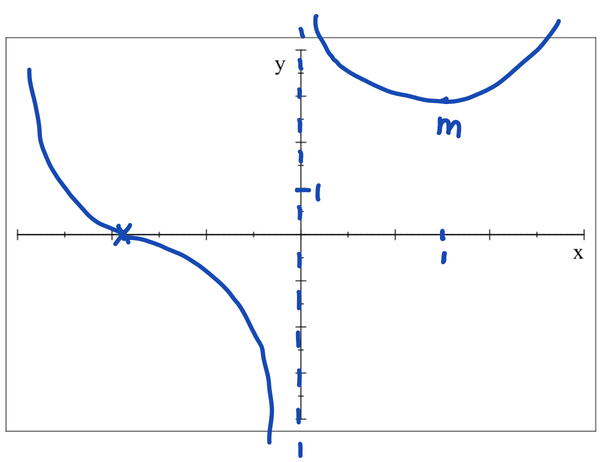
9.

a. No horizontal asymptote, and one vertical asymptote $x = 0$.

b. Increasing on $(1, \infty)$, decreasing on $(-\infty, 0) \cup (0,1)$, and one relative minimum at $(1,3)$.

c. Concave up on $(-\infty, -\sqrt[3]{2}) \cup (0, \infty)$, concave down on $(-\sqrt[3]{2}, 0)$, and one inflection point at $(-\sqrt[3]{2}, 0)$.

d.



10.

a. $c = \sqrt[3]{9}$

b. $c = 2.25$

11. $dV = 0.8\pi\ in^3$

12. -68

13. $-23\cot x+\frac{3}{\sqrt{2}}\sec x+C$

14. 23.25

15. $\sim 4.327\ moles/L$

16. $s(t)=3e^{t}+2\cos t-5$

17. $F'(x)=\arcsin((3x+5)^{2})\cdot 2(3x+5)(3)$

18. $A\sim 6.798\ units^{2}$