DAWSON COLLEGE
Mathematics Department
FINAL EXAMINATION 201-NYA-05 (Calculus I – Science)
WEDNESDAY DECEMBER 22nd, 2021

(1) **(5+5+5 marks)** Find the limit if it exists. If the limit does not exist, explain why.

Do not use L'Hôpital's Rule.

(a) $\lim_{x\to 1} \dfrac{x^3-1}{3x^2+2x-5}$

(b) $\lim_{x\to 0^+} \dfrac{\sin\left(4x^2\right)}{\tan\left(\sqrt{3x}\right)}$

(c) $\lim_{x\to 0^+} \dfrac{4e^{\frac{1}{x}}}{3e^{\frac{1}{x}}+2}$

(2) **(5 marks)** Use only the limit definition of the derivative to calculate $f'(x)$

if $f(x)=\sqrt{1-2x}$.

(3) **(2 marks)** Find the value that $\lim_{x\to 0} g(x)$ must have if the given limit statement holds:

$$\lim_{x\to 0} \frac{5-g(x)}{x+4} = 1$$

(4) **(5 marks)** Given:

$$f(x)=\begin{cases}\dfrac{x\left(x^2+3x-10\right)}{(x+5)^2} & x\le 0\\ x\sin\left(\dfrac{5}{x^2}\right) & x>0\end{cases}$$

(a) State the definition of continuity of a function at a point.

(b) Using this definition find the point(s) of discontinuity of the function above and specify their types.

(5) **(4+4+4+4 marks)** Find $f'(x)$ if:

(Do not simplify your answers)

(a) $f(x)=\left(\dfrac{4x+\sqrt[3]{x}}{3x^2-x+1}\right)^3$

(b) $f(x)=\sin\left(x^2\tan x\right)+2^{\sqrt{\cot(4x)}}$

(c) $f\left(x\right)=\sqrt{1-4x^2}\arccos(2x)$

(d) $f\left(x\right)=\left(1+\ln x\right)^{\sec x}$

(6) **(3+2 marks)** If $\sin\left(x^2+y^2\right)+3x\tan y=5y$ then find:

(a) $\frac{dy}{dx}=y'$

(b) an equation of the tangent line to the curve at point (0,0).

(7) **(3 marks)** Find the value(s) of x for which the function $y = 4^x$ verifies the equation: $y'' + (\ln 4)\, y' = 32(\ln 4)^2$.

(8) (**5+5 marks)** Evaluate, **using L'Hôpital's Rule,** the following limits:

(a) $\lim_{x \to 0} \dfrac{\sin x - x \cos x}{x - \sin x}$

(b) $\lim_{x\to 0}\left(\cos x\right)^{\frac{5}{x^2}}$

(9) **(5 marks)** The sides of an equilateral triangle (a triangle with all sides equal) are increasing at the rate of $0.3cm/hr$. At what rate is the area of the triangle changing when the side length is $5cm$?

(10) **(1+1 marks)** Decide if the following statements are true or false. Give a justification or a (counter)example.

(a) If f is not continuous on $[a,b]$, it cannot have both an absolute maximum and an absolute minimum on $[a,b]$.

(b) If f is not differentiable at $x=a$ then f has a vertical asymptote at $x=a$.

(11) **(12 marks)** If $f(x)=\dfrac{-x}{\left(x^2-1\right)^2}$ and $f'(x)=\dfrac{3x^2+1}{\left(x^2-1\right)^3}$. $f''(x)=\dfrac{-12x\left(x^2+1\right)}{\left(x^2-1\right)^4}$

then:

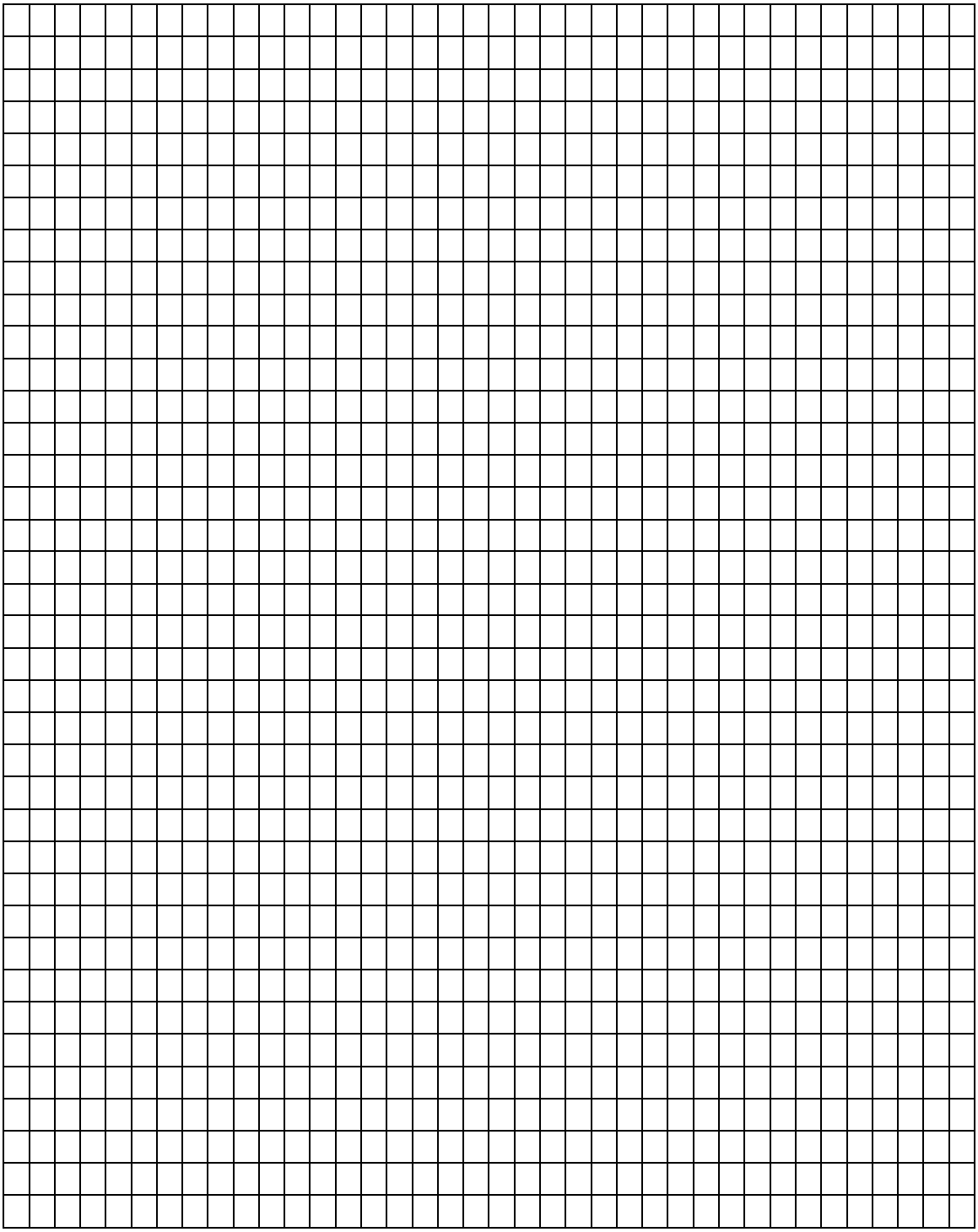
(a) Find the x-intercepts and the y-intercept, if any.

(b) Find all the horizontal and vertical asymptotes, if any. Justify your answers using limits.

(c) Find the intervals where the function is increasing and where it is decreasing. Find the points of local minimum and local maximum, if any.

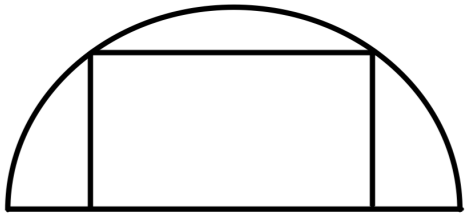
(d) Find the intervals where the function is concave upward and concave downward. Find the inflection points, if any.

(e) Sketch the graph of the function and clearly label all the points on the graph.



(12) **(5 marks)** A rectangle is to be inscribed in a semicircle of radius 2 cm. What is the largest area the rectangle can have, and what are its dimensions?

(Note: The equation of a circle with center at the origin and radius r is $x^2 + y^2 = r^2$)



(13) **(5+5 marks)** Find:

(a) $\int\left(\csc^2 x+\frac{3x}{2\sqrt{x}}+5e^x\right)dx$

(b) $\int\frac{\cos\left(\sqrt{x}\right)}{5\sqrt{x}}dx$

(14) **(5 marks)** Given the conditions $f''(x) = x(3+x)^2, f(0) = -1, f(1) = 2$ then find $f(x)$.

Answers:

1. (a) $\frac{3}{8}$

   (b) 0

   (c) $\frac{4}{3}$

2. $\frac{-1}{\sqrt{1-2x}}$

3. 1

4. $x=-5$ infinite discontinuity

5. (a) $3\left(\frac{4x+\sqrt[3]{x}}{3x^2-x+1}\right)^2 \cdot \frac{\left(4+\frac{1}{3}x^{-\frac{2}{3}}\right)(3x^2-x+1)-(4x+\sqrt[3]{x})(6x-1)}{(3x^2-x+1)^2}$

   (b) $\cos(x^2\tan x)\cdot(2x\tan x + x^2\sec^2 x) + 2^{\sqrt{\cot(4x)}}\cdot\ln 2\cdot\frac{1}{2}(\cot(4x))^{-\frac{1}{2}}\cdot(-\csc^2(4x))\cdot 4$

   (c) $\frac{1}{2}(1-4x^2)^{-\frac{1}{2}}(-8x)\arccos(2x) + \sqrt{1-4x^2}\cdot\frac{-1}{\sqrt{1-4x^2}}$

   (d) $(1+\ln x)^{\sec x}\left(\sec x\tan x\ln(1+\ln x) + \frac{\sec x}{x(1+\ln x)}\right)$

6. (a) $\frac{-3\tan y - 2x\cos(x^2+y^2)}{2y\cos(x^2+y^2)+3x\sec^2 y - 5}$

   (b) $y=0$

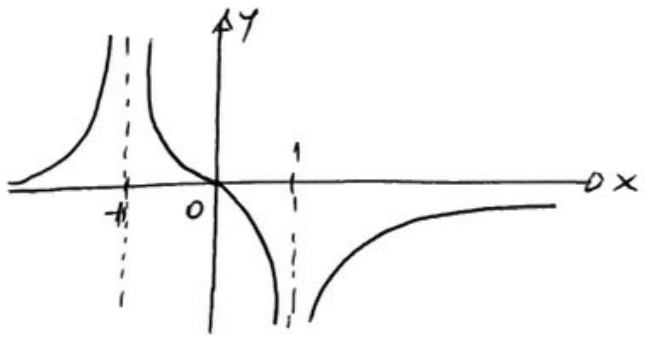
7. $x=2$

8. (a) 2 (b) $e^{-\frac{5}{2}}$

9. $\frac{1.5\sqrt{3}}{2}\ cm^2/hr$

10. (a) False (b) False

11.



12. $A = 4$

$x = 2\sqrt{2}$
$y = \sqrt{2}$
} dimensions

13. (a) $-\cot x + x^{\frac{3}{2}} + 5e^x + C$

(b) $\frac{2}{5}\sin(\sqrt{x}) + C$

14. $f(x) = \frac{x^5}{20} + \frac{x^4}{2} + \frac{3x^3}{2} + \frac{19x}{20} - 1$